Structure-Preserved Power System Transient Stability Using transient Energy Functions

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***Abstract*— It is very important to maintain supply reliability under the deregulated environment. The transient stability problem is one of the major concerns in studies of planning and operation of power systems. Although the equal-area criterion method is useful in determining the stability as a transient stability evaluation method, the method is only applicable to a one-machine system connected to an infinite bus or to a two machine system and the time domain simulation is the best available tool for allowing the use of detailed models and for providing reliable results. The main limitations of this approach involve a large computation time. This paper describes a method for estimating a normalized power system transient stability of a power system that is SMIB and three machines, nine bus systems. The energy function is derived using a Center of Inertia (COI) formulation. The critical energy is evaluated using corresponding energy function Therefore, the transient energy function (TEF) is constructed for large power system**

***Index Terms*—Critical clearing time, direct stability analysis, multi-machine system, transient energy function (TEF).**

I. INTRODUCTION

An interconnected power system consists of generating units run by prime-movers (including turbine-governor and excitation control systems) plus transmission lines, loads, transformers, static reactive compensators, and high-voltage direct-current lines. The size of the interconnection varies depending on the system but the technical problems are the same. At the planning level, the planner would invariably study the stability of the system for a set of disturbances ranging from a three-phase-to-ground fault (whose probability of occurrence is rare) to single-phase faults, which constitute about 70 percent of the disturbances. The planner desires to determine if a potential fault has an adequate margin of safety without the system losing synchronism. A system is said to be synchronously stable (i.e., retain synchronism) for a given fault if the system variables settle down to some steady-state values as time approaches infinity after the fault is removed. These simulation studies are called *transient stability studies*. Transient stability is one of the important items which should be investigated in power system planning and its operation. Present day transient stability analyses are mainly performed by simulations. This method is very reliable method, but it does not suit calculations of many cases because it takes much computing time. As a substitute, direct method was proposed, and many papers have been reported for this method. It has reached to some level for a simple model in which generators are represented by constant voltages behind transient reactances.

Some energy functions describe the system transient energy using a synchronous frame of reference [1]-[3].Others, as in the approach proposed here, has used a center of inertia (COI.) formulation [4]-[7]. Lyapunov's or energy functions, is the method being implemented for assessment of online dynamic security.

Transient stability analysis programs are MATLAB, PSCAD, ETAP, etc.. In these simulation programs, the behavior of a power system is evaluated to determine its stability and/or its operating limits, or eventually, in order to determine the need for additional facilities. Important decisions are made based only on the results of stability studies. It is therefore important to ensure that the results of stability studies are as timely and accurate as possible. Thus, it is important for a power system to remain in a state of operating equilibrium under normal operating conditions as well as during the presence of a disturbance.

The main purpose of this paper is to investigate the transient stability of the SMIB and three machine nine bus power system with energy function method, when subjected to large disturbances .

The transient energy consists of two components: kinetic and potential energy. In the post-disturbance period, profiles of the kinetic energy (V_{KE}), the potential energy (V_{PE}) are obtained. These are used to develop a criterion for the degree of stress on a disturbed but stable machine, and to assess the extent of instability for an unstable machine.

In practice, CCT can be obtained in one of two ways: by trial and error analysis of system post disturbance equations [1]-[2] or by integrating fault-on equations and checking the value of Lyapunov's energy function until it reaches a previously determined critical level [3]. For the first approach, many integration processes are necessary. But, for the second approach we can evaluate the CCT in just one integration process.

The IEEE 3-machine 9-bus [4] test systems are used to illustrate the proposed approach in the CCT evaluation.

II. MATHEMATICAL FORMULATION

In its simplest form the transition of a power system undergoing a disturbance is described by a set of three differential equations

$$\dot{X}(t) = f^{I}(X(t)) \qquad -\infty < t \le 0 \qquad (1)$$

$$\dot{X}(t) = f^{F}(X(t)) \qquad 0 < t \le t_{cl} \qquad (2)$$

$$\dot{X}(t) = f(X(t)) \qquad t_{cl} < t \le \infty \qquad (3)$$

X (t) is the vector of state variables of the system at time t. At t = 0, a fault occurs and the dynamics change from f^I to f^F. During 0 < t ≤ tcl, called the faulted period, the system is governed by fault- on dynamics $f^{F.}$ f^F indicates that there are no structural changes between t=0 and t = t_{cl}. When the fault is cleared at t= tcl, we have the post fault dynamics f(X (t)).In the prefault state -∞ < t ≤ 0, the system would have settled down to a steady state so that X (0) =X0 is known. Therefore we have the model as

$$\dot{X}(t) = f^{F}(X(t)) \qquad 0 < t \le t_{cl} \qquad (4)$$

$$X(0) = X_0$$

and

$$\dot{X}(t) = f(X(t)) \qquad t > t_{cl} \qquad (5)$$

In reality the model is a set of differential – algebraic equations, i.e. for a dynamic system

$$\dot{X} = f^{F}(x(t), y(t))$$

$$0 = g^{F}(x(t), y(t)) \qquad (6)$$

and

$$\dot{X} = f(x(t), y(t))$$

$$0 = g(x(t), y(t)) \qquad (7)$$

Instead of finding the numerical solutions of DAE for a given time period the transient stability can be assessed directly through Lyapunov's direct method of stability. Lyapunov's stability theorem is stated as following

$$\dot{X} = f(X) \qquad (8)$$

If there exist a positive definite continuous function *V (X)*, whose first partial derivative with respect to the state variable exist, then if the total derivative *V(X)* is negative semi definite then the system is said to be stable. The function *V (X)* is called as Lyapunov's energy function. In [6], a method was proposed to estimate the transient stability of a system using Transient energy function.

III. ENERGY FUNCTION FOR A SINGLE MACHINE INFINITE BUS SYSTEM

The energy function is always constructed for the post fault system. Thus the post fault equations is

$$M\frac{d^2\delta}{dt^2} = P_m - P_e^{\max}\sin\delta \qquad (9)$$

Where

$$P_e^{\max} = \frac{E_1 E_2}{X}$$

E_1 = Transient internal voltage

E_2 = Infinite bus bar voltage

X = Transfer reactance before the fault

δ = Generator rotor angle deviation in radians

P_m = mechanical power input

The right hand side of Equation (9) can be expressed as the negative gradient of a potential energy function V_{PE} i.e.

$$M\frac{d^2\delta}{dt^2} = -\frac{\partial V_{PE}(\delta)}{\partial\delta} \qquad (10)$$

Where

$$V_{PE}(\delta) = -P_m\delta - P_e^{\max}\cos\delta \qquad (11)$$

Multiplying Equation (10) by $\frac{d\delta}{dt}$ on both sides and Integrating, we get

$$\frac{d}{dt}\left(\frac{1}{2}M\left(\frac{d\delta}{dt}\right)^2+V_{PE}\left(\delta\right)\right)=0 \tag{12}$$

Since $\frac{d\delta}{dt}=\omega$, this implies $\frac{dV}{dt}=0$ where

$$V\left(\delta,\omega\right)=\frac{1}{2}M\omega^2+V_{PE}\left(\delta\right)+C \tag{13}$$

With C as a constant of integration. The constant C is adjusted so that $V\left(\delta^s,0\right)=0$. Therefore, $C=-V_{PE}\left(\delta^s\right)$.

By substituting value of C in the Equation (3.5), then equation will be

$$V\left(\delta,\omega\right)=\frac{1}{2}M\omega^2+V_{PE}\left(\delta\right)-V_{PE}\left(\delta^s\right) \tag{14}$$

The post fault equilibrium point is given by

$$\delta^s=\sin^{-1}\left(\frac{P_m}{P_e^{\max}}\right) \tag{15}$$

The nearest unstable equilibrium point is given by

$$\delta^u=\pi-\delta^s \tag{16}$$

Therefore,

$$V_{PE}\left(\delta^s\right)=P_e^{\max}(\cos\delta-\cos\delta^s) \tag{17}$$

Sub Equation (3.3) and Equation (3.9) in the Equation (3.6), then $V\left(\delta,\omega\right)$ is given by

$$V\left(\delta,\omega\right)=\frac{1}{2}M\omega^2-P_m\left(\delta-\delta^s\right)-P_e^{\max}\left(\cos\delta-\cos\delta^s\right) \tag{18}$$

In the case of a single machine system, V_{cr} is determined as $V_{cr}=V\left(\delta^u,0\right)$, i.e.

$$V_{cr}=-P_m\left(\delta^u-\delta^s\right)-P_e^{\max}\left(\cos\delta^u-\cos\delta^s\right) \tag{19}$$

Since $\delta^u=\pi-\delta^s$, we get V_{cr} as

$$V_{cr}=-P_m(\pi-2\delta^s)+2P_e^{\max}\cos\delta^s \tag{20}$$

The energy function is given by

$$\begin{aligned}V\left(\delta,\omega\right)&=\frac{1}{2}M\omega^2-P_m\left(\delta-\delta^s\right)-P_e^{\max}\left(\cos\delta-\cos\delta^s\right)\\&=V_{KE}+V_{PE}(\delta,\delta^s)\end{aligned} \tag{21}$$

Where

$V_{KE}=\frac{1}{2}\omega^2$ is the transient kinetic energy

$V_{PE}\left(\delta,\delta^s\right)=-P_m\left(\delta-\delta^s\right)-P_e^{\max}\left(\cos\delta-\cos\delta^s\right)$ is the potential energy.

The system is stable for t=t_{cl}, if along the faulted trajectory $V^{cl}\left(\delta^{cl},\omega^{cl}\right)<V_{cr}$ at t = t_{cl} and the critical clearing time t_{cr} is obtained when $V\left(\delta,\omega\right)=V_{cr}$ on the faulted trajectory. The energy function is labelled as because this is the maximum energy that the system can have without becoming unstable. If the energy exceeds this critical energy then the system is unstable. Hence, the system stability can be assessed by computing the transient energy at critical clearing angle and checking if it less than that is

$$V_{cl}\left(\delta_c,\omega\right)<V_{cr}\left(\delta_u,\omega\right) \qquad \textit{stable}$$

$$V_{cl}\left(\delta_c,\omega\right)>V_{cr}\left(\delta_u,\omega\right) \qquad \textit{unstable}$$

IV. ENERGY FUNCTION FOR A MULTI MACHINE SYSTEM

In a similar way the transient stability of multi-machine can be assessed through transient energy function method [7]. Let the swing equation of an *ith* generator be given as

$$\begin{aligned}M_i\dot{\tilde{\omega}}_i&=P_{mi}-P_{ei}\\ \dot{\theta}_i&=\tilde{\omega}_i \qquad i=1,\dots,n\end{aligned} \tag{22}$$

If we eliminate all the terminal buses and load buses, except the generator internal nodes, then

$$I_G=Y_{red}E_G \tag{23}$$

Where,

Y_{Red} is the reduced admittance matrix consisting self and transfer admittances of the internal generator nodes.

The vector of internal voltages is represented as $\mathbf{E}_G$ and the vector of generator currents as $\mathbf{I}_G$.

The real power output of an i^{th} generator is given as

$$\begin{aligned}P_{ei}&=\mathrm{Re}(E_{Gi}(I_{Gi})^*)\\&=\mathrm{Re}(E_{Gi}(Y_{red}E_G)^*)\\&=E_{Gi}G_{ii}+\sum_{\substack{k=1\\ \neq i}}^{n}E_{Gi}E_{Gk}(G_{ik}Cos\delta_{ik}+B_{ik}Sin\delta_{ik})\end{aligned} \tag{24}$$

Let, $P_i=P_{mi}-E_{Gi}{}^2G_{ii}$ then

$$M_i\frac{d^2\delta_i}{dt^2}=\left(P_i-\sum_{\substack{k=1\\ \neq i}}^{n}E_{Gi}E_{Gk}(G_{ik}Cos\delta_{ik}+B_{ik}Sin\delta_{ik})\right)$$

(25)

Defining all the rotor angles and speed in terms of Centre of Inertia (COI) as

$$\delta_0 = \frac{1}{M_T}\sum_{i=1}^{n} M_i \delta_i$$

$$\omega_0 = \frac{1}{M_T}\sum_{i=1}^{n} M_i \omega_i \qquad (26)$$

Where

$$M_T = \sum_{i=1}^{n} M_i$$

We then transform the variables $\delta_{i,}\omega_i$ to the COI variables as

$$\theta_i = \delta_i - \delta_0$$

$$\tilde{\omega}_i = \dot{\theta}_i = \omega_i - \omega_0$$

It is easy to verify

$$\dot{\theta}_i = \dot{\delta}_i - \dot{\delta}_0$$
$$= \omega_i - \omega_0$$
$$\square\ \tilde{\omega}_i$$

Equation (25) can be represented in terms of COI variables as

$$M_i \frac{d^2\theta_i}{dt^2} + M_i \frac{d^2\delta_0}{dt^2} = P_i - \sum_{\substack{k=1 \\ \neq i}}^{n} \left(C_{ik}\sin(\theta_{ik}) + D_{ik}\cos(\theta_{ik})\right) \qquad (27)$$

$$M_i \frac{d^2\delta_o}{dt^2} = \frac{M_i}{M_T}\sum_{i=1}^{n} M_i \frac{d^2\delta_i}{dt^2}$$

$$= \frac{M_i}{M_T}\sum_{i=1}^{n}\left(P_i - \sum_{\substack{k=1 \\ \neq i}}^{n} E_{Gi}E_{Gk}(G_{ik}Cos\theta_{ik} + B_{ik}Sin\theta_{ik})\right)$$

$$= \frac{M_i}{M_T}\left(\sum_{i=1}^{n} P_i - \sum_{i=1}^{n}\sum_{\substack{k=1 \\ \neq i}}^{n} E_{Gi}E_{Gk}(G_{ik}Cos\theta_{ik} + B_{ik}Sin\theta_{ik}\right)$$

$$= \frac{M_i}{M_T}P_{COI} \qquad (28)$$

Hence equation (27) can be written as the swing equation with $D_i=0$ as

$$M_i\dot{\tilde{\omega}}_i = P_i - P_{ei} - \frac{M_i}{M_T}P_{COI}\ \square\ f_i(\theta) \qquad i = 1,\dots,n$$

$$\frac{d\theta_i}{dt} = \tilde{\omega} i$$

(29)

Where

$$P_{COI} = \sum_{i=1}^{n} P_i - P_{ei}$$

The right-hand side in Eq. (29) has different parameters (i.e., G_{ik} and B_{ik} values) in computing Pei and P_{COI} for faulted period (0 < t ≤ tcl) and the post-fault period (t > tcl).

The energy function is given by

$$V(\theta,\tilde{\omega}) = \frac{1}{2}\sum_{i=1}^{n} M_i\tilde{\omega}_i^2 - \sum_{i=1}^{n}\int_{\theta_i^s}^{\theta_i} f_i(\theta)d\theta_i \qquad (31)$$
$$= V_{KE}(\tilde{\omega}) + V_{PE}(\theta)\ \square\ V_{TOT}$$

Where Θ_i and $\omega_{\square i}$ are the variables from the faulted trajectory.

Let the post fault system given by (30) have the stable equilibrium point at $\theta = \theta^s, \tilde{\omega} = 0$. Where, θ_i^s is the i^{th} generator rotor angle in COI. The function f_i (θ) is given as

$$f_i(\theta) = P_i - P_{ei} - \frac{M_i}{M_T}P_{COI} = 0 \qquad i = 1.,\dots..n \qquad (32)$$

Summing the energy function defined in (31) for all the generators lead to

$$V(\theta,\tilde{\omega}) = \frac{1}{2}\sum_{i=1}^{n} M_i\tilde{\omega}_i^2 - \sum_{i=1}^{n} P_i\left(\theta_i - \theta_i^s\right) - \sum_{i=1}^{n-1}\sum_{k=i+1}^{n} C_{ik}\left(Cos\theta_{ik} - Cos\theta_{ik}^s\right)$$
$$- Dij\frac{\left(\theta_i - \theta_i^s\right) + \left(\theta_k - \theta_k^s\right)}{\left(\theta_i - \theta_i^s\right) - \left(\theta_k - \theta_k^s\right)}\left(\sin\theta_{ik} - \sin\theta_{ik}^s\right) \qquad (34)$$

Where $\frac{1}{2}\sum_{i=1}^{n} M_i\tilde{\omega}_i^2$ is the change in rotor kinetic energy of all the generators in COI reference frame

The rest of the terms are nothing but the potential energy of the system.

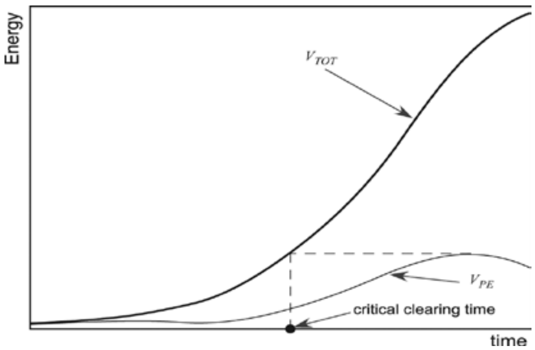

Fig. 1 Total energy versus the potential energy. The critical clearing time is the time at which the total energy equals the maximum potential energy

Now compute the energy function given in (34) for a clearing angle and if the energy is less than the critical energy then the system is stable else the system is not stable.

V. SIMULATION AND RESULTS

Test System for SMIB system

The validity of the proposed method is shown by Simulation studies. For the simulation studies, we use the single machine infinite bus system shown in Fig. 2.

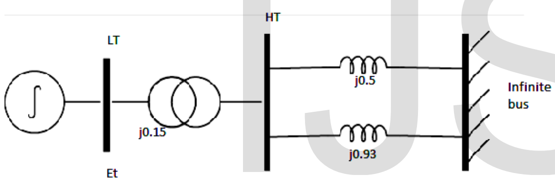

Fig. 2 Small test system.

Simulation Results

Time Vs Rotor Angle

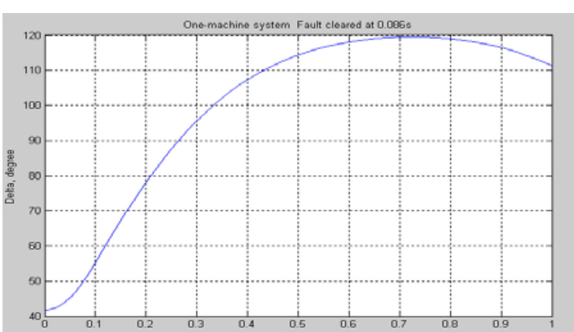

Fig .3 Time Vs Rotor Angle at fault clearing time 0.086 sec

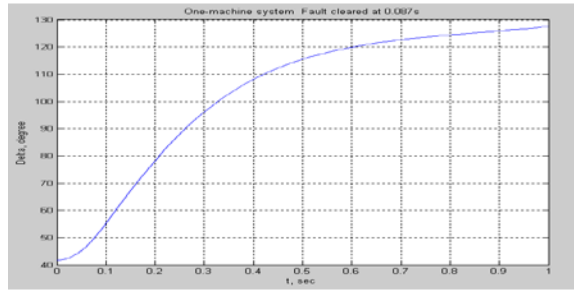

Fig 4 Time Vs Rotor Angle at fault clearing time 0.087 sec, system is unstable after fault clearing time

TABLE I
TOTAL ENERGY COMPARISON

Faulted bus	Tcr sec	Critical energy (Vcr)	Total energy V(δ.ω)
2	0.086	0.1651	0.1651

From the table I, we see that the system is stable with t_{cr}=0.086 sec and system becomes unstable after this tcr. Therefore the critical clearing time is 0.086 sec. this is proved when critical energy (Vcr) is equal to the total energy V (δ, ω) , therefore critical clearing time at this point is 0.086sec as Vcr is equal to V(δ,ω)

TABLE II
CCT CALCULATION

Faulted bus	Critical Clearing Time t_{cr}(sec)	
	Energy Comparison from [2]	Numerical Method
2	0.086	0.07 – 0.087

In Table II, the proposed method is used to determine the CCT and compare the result with step-by-step integration as a benchmark and the results obtained by using methods of References [2] and [7].

Test System for Multi machines system

The validity of the proposed method is shown by Simulation studies. For the simulation studies, we use the three machine nine bus system shown in Fig. 5.

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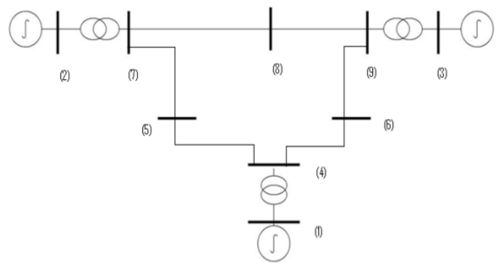

Fig 5 Three-machine, nine-bus test system

Simulation Result

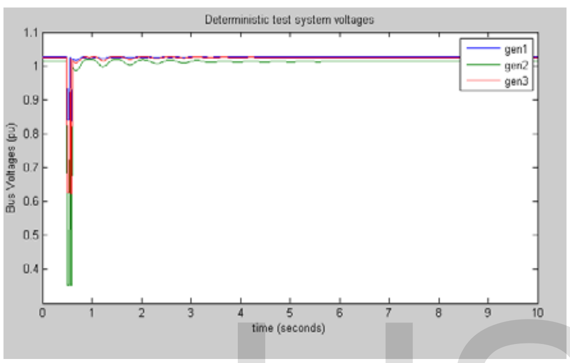

Fig 6 Deterministic test system voltages.

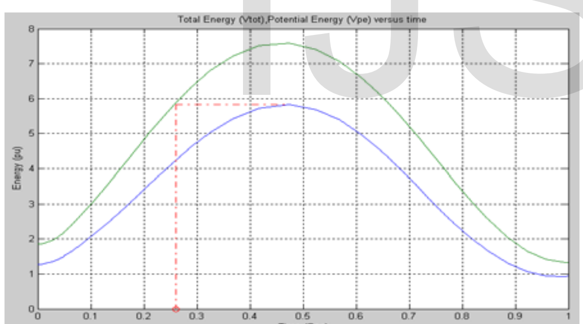

Fig. 7 Total energy versus the potential energy. The critical clearing time is the time at which the total energy equals the maximum potential energy Critical Clearing Time = 0.2602 sec

TABLE III
DETERMINING CRITICAL CLEARING TIME USING PROPOSED METHOD

Faulted Bus	Removed Lines	tcr (sec) (proposed method)	tcr (sec) (numerical method)	Vcr= V, then System is stable
7	5,7	0.262	0.2	5.880

The critical clearing time is the time at which the total energy equals the maximum potential energy. Critical Clearing Time = 0.2602 sec
From Table III, the proposed method gives fairly consistent results compared to the numerical integration method. The critical clearing time from the proposed method is very close to the critical clearing of the numerical integration.

V. Concluding Remarks

This paper develops an approach to analyze the impact of random load and generation variations on the transient stability of a structure preserved power system. The well-known energy function method for power system transient stability is used as a basis to explore the power system stability through a Lyapunov's stability analysis.

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